Joint Real-Time Computation of Allan Deviation, Time Deviation, and Hadamard Deviation

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INTRODUCTION

Allan deviation ADEV and time deviation TDEV allow the variations of time interval provided by the synchronization signal to be assessed and the type of phase noise affecting the signal to be recognized. The parameters are commonly used for describing the quality of synchronization signal in the telecommunication network [1, 2, 3]. Hadamard deviation HDEV is insensitive for the presence of frequency drift and can be used for recognition of higher types of phase noises. The estimates of the parameters are computed for a series of observation intervals using the sequence of time error samples previously measured at some network interface. The evaluation of the synchronization signal is commonly a two-stage process: calculation of a parameter follows the time error measurement. It causes an obvious delay of the evaluation process. The application of real-time computation of the parameters allows to reduce the time of the analysis.

Authors of the paper have already proposed the methods of real-time assessment of Allan deviation and time deviation [6] as well as the method of joint real-time assessment of both parameters [7]. These methods allow to compute the estimates of ADEV and TDEV (which is characterized by more complex estimator's formula) in the real-time, during the measurement process, simultaneously for a set of observation intervals. The results of experimental tests presented in [6, 7] have proved the ability of the real-time computation of Allan deviation and time deviation. Rather small maximum time used for the computation within one sampling interval was obtained also for joint computation of ADEV and TDEV. Including the Hadamard deviation HDEV to the timing signal evaluation process could give us additional useful possibilities of timing signal analysis.

In this paper the method of real-time assessment of the Hadamard deviation is presented. The rearrangement of the HDEV estimator's formula is performed. Then the organization of the real-time computation process for the set of observation intervals is described. The method of the real-time HDEV computation performed jointly with TDEV or/and ADEV is also presented.

In order to calculate the parameters' estimates simultaneously for several observation intervals in the real time, all necessary operations should be performed in the time period between two sampling instants, i.e. during the sampling interval τ_0. For a given observation interval $\tau=n\tau_0$ and current sampling instant, the rearrangement of TDEV computation, described in [6], enables the sample measured at the current instant and the samples measured n, $2n$, and $3n$ (for TDEV) sampling instants earlier to be used for ADEV as well as TDEV computation. The same samples can be used for HDEV computation. Therefore, the influence of the critical issue – time of access to data values measured earlier and stored in the equipment's memory – is minimized.

In the paper the results of experimental tests of the method proposed for different conditions are presented. The calculations were performed for the time error sequences taken with different sampling interval τ_0=1/30 s, which enables to compute the parameters for the series of observation intervals starting from τ_{min}=0.1 s. Different numbers, lengths, and ranges of the observation intervals simultaneously analyzed were considered. The results of computation obtained using different computers are presented and compared.

ADEV, TDEV, AND HDEV ESTIMATORS

Allan deviation and time deviation are computed based on the averaging of second differences of the phase process $x(t)$ of the analyzed timing signal. We can assume for the telecommunication applications, in the case of negligible influence of frequency drift, that ADEV and TDEV are estimated based on the time error function measured between the analyzed timing signal and the reference one [4]. In the case of Hadamard deviation the third difference of the phase process $x(t)$ is averaged.

The formulae for the estimators of Allan deviation ADEV, time deviation TDEV, and Hadamard deviation take the form:

$$A\hat{D}EV(\tau)=\sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}(x_{i+2n}-2x_{i+n}+x_i)^2} \tag{1}$$

$$T\hat{D}EV(\tau)=\sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{j+n-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2} \tag{2}$$

$$H\hat{D}EV(\tau)=\sqrt{\frac{1}{6n^2\tau_0^2(N-3n)}\sum_{i=1}^{N-3n}(x_{i+3n}-3x_{i+2n}+3x_{i+n}-x_i)^2} \tag{3}$$

where $\{x_i\}$ is a sequence of N samples of time error function $x(t)$ taken with interval τ_0; $\tau=n\tau_0$ is an observation interval. For TDEV computation the estimator formula (2) can be changed in order to simplify the calculation of sum [4, 5] and takes the form:

$$T\hat{D}EV(n\tau_0)=\sqrt{\frac{1}{6}\cdot\frac{1}{N-3n+1}\cdot\frac{1}{n^2}\sum_{j=1}^{N-3n+1}S_j^2(n)} \tag{4}$$

where

$$S_j(n)=S_{j-1}(n)-x_{j-1}+3x_{j+n-1}-3x_{j+2n-1}+x_{j+3n-1} \tag{5}$$

$$S_1(n)=\sum_{i=1}^{n}(x_{i+2n}-2x_{i+n}+x_i) \tag{6}$$

When computing in the real time, we do not have access to the time error samples indexed by $i+n$ or $i+2n$ for the current time instant described by index i, because these samples have not been measured yet. We have access to the sample currently measured (for the current sampling instant i) and the samples measured earlier (with indexes smaller than i) and stored in the equipment's memory. The schemes of indexing the samples for off-line and real-time computation are presented in Fig. 1 and Fig. 2, respectively. Therefore, the indexes in formulae for ADEV, TDEV, and HDEV estimators should be changed in the case of real-time calculation.

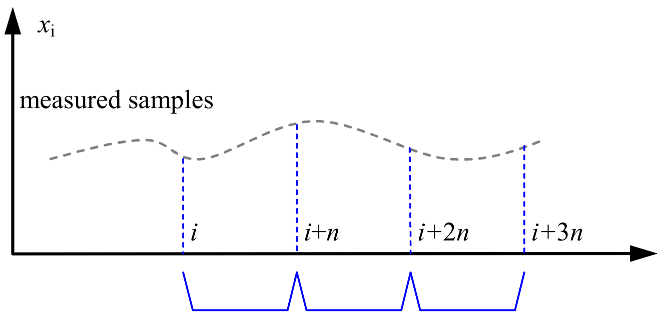

Fig. 1. Samples' indexing for off-line computation

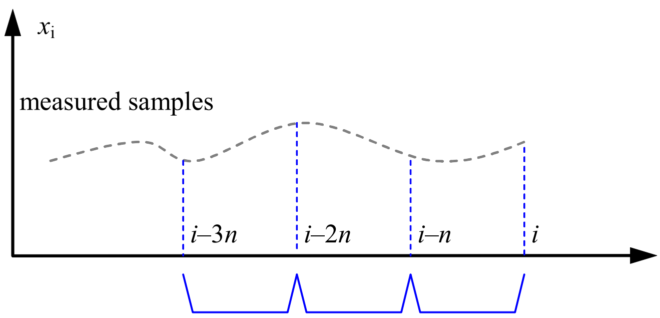

Fig. 2. Samples' indexing for real-time computation

The rearrangement of indexes for the estimators of ADEV and TDEV was performed in [6]. As a result we have obtained the ADEV estimator's formula for a current instant i in the form depending of the sum of squares of second differences computed for the previous sampling instant $i-1$

$$A\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{2n^2\tau_0^2(i-2n)}\left(A_{i-1}(n)+\left(x_i-2x_{i-n}+x_{i-2n}\right)^2\right)} \tag{7}$$

were $A_i(n)$ is the sum of squares of second differences of time error samples

$$A_i(n) = \sum_{j=2n+1}^{i}\left(x_j-2x_{j-n}+x_{j-2n}\right)^2,\quad i>2n \tag{8}$$

The conversion of the time deviation estimator given in the simplified form (4-6) brought the formula dependent on the overall sum of squares and internal sum computed for the instant $i-1$ and four time error samples

$$T\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6}\cdot\frac{1}{i-3n+1}\cdot\frac{1}{n^2}\left[S_{ov,i-1}(n)+\left(S_{i-1}(n)-x_{i-3n}+3x_{i-2n}-3x_{i-n}+x_i\right)^2\right]} \tag{9}$$

where $S_{ov,i}(n)$ is the overall sum updated for each sample i, given in the form:

$$S_{ov,i}(n) = S_{ov,i-1}(n)+S_i^2(n) \tag{10}$$

where

$$S_i(n) = S_{i-1}(n)-x_{i-3n}+3x_{i+2n}-3x_{i+n}+x_i,\quad i>3n \tag{11}$$

and

$$S_{3n}(n) = \sum_{j=2n+1}^{3n}\left(x_j-2x_{j-n}+x_{j-2n}\right),\quad j>2n \tag{12}$$

The sum $S_{3n}(n)$ is the first element of the overall sum $S_{ov,i}(n)$ given by (10). Similar procedure should be performed for the HDEV estimator. The formula for the estimator of Hadamard deviation, for a current index i of measured sample and the observation interval $\tau=n\tau_0$, takes the form

$$H\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6n^2\tau_0^2(i-3n)}\sum_{j=3n+1}^{i}\left(x_j-3x_{j-n}+3x_{j-2n}-x_{j-3n}\right)^2} \tag{13}$$

Presenting the sum of squares of third differences in the form

$$B_i(n) = \sum_{j=3n+1}^{i}\left(x_j-3x_{j-n}+3x_{j-2n}-x_{j-3n}\right)^2 \tag{14}$$

we can write the equation (14) in the form

$$H\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6n^2\tau_0^2(i-3n)}B_i(n)} \tag{15}$$

Taking into consideration the sum of squares computed for a preceding sample, the operations of HDEV computation for i-th sampling interval are performed using the formula:

$$H\hat{D}EV_i(n\tau_0) = \sqrt{\frac{1}{6n^2\tau_0^2(i-3n)}\left(B_{i-1}(n)+\left(x_i-3x_{i-n}+3x_{i-2n}-x_{i-3n}\right)^2\right)} \tag{16}$$

As a result of the conversion of the parameters' formulae, in order to compute ADEV, TDEV, and HDEV for a current sampling instant i and given observation interval $\tau=n\tau_0$, we need the values of appropriate sum $A_{i-1}(n)$, $S_{ov,i-1}(n)$, $S_{i-1}(n)$, and $B_{i-1}(n)$, currently measured sample x_i and the samples x_{i-n}, x_{i-2n}, and x_{i-3n} previously measured and stored in memory.

PROCEDURE OF REAL-TIME COMPUTATION

The formulae of the parameters given in the forms presented in the previous section allow us to perform the computation in the real time, during the time error measurement process. The calculation can be performed jointly for all parameters considered, as well as for pairs of parameters or for single parameter. A general procedure of the real-time computation of HDEV jointly with ADEV or TDEV is similar to the procedure of joint ADEV and TDEV real-time computation presented in [7]. The course of operations is as follows:

1. Measure a new time error sample and store it in a data file.

2. Compute the appropriated differences for a given n (observation interval $\tau = n\tau_0$) using the current sample, and the samples measured n, $2n$ or $3n$ sampling intervals earlier.
3. Update the appropriated sums.
4. Compute current averages and their square roots.
5. Execute Steps 2-4 for successive larger observation intervals (larger n).
6. Return to Step 1 (measure a new sample).
7. When the measurement is finished, the values of the parameter's estimate for the observation intervals considered are known.

The computations of ADEV and TDEV start when the sample no. $2n+1$ has been measured. The first value of ADEV estimate can be computed at this instant. However, for the TDEV the computation of the internal sum $S_i(n)$ only just starts. The first values of TDEV and HDEV can be computed after the sample no. $3n+1$ has been measured.

An example of the procedure of real-time computation of HDEV, TDEV, and ADEV for single observation interval $3\tau_0$ performed jointly is presented in Figs. 3 to 5. The early stage of the process is presented in Fig. 3. Only four time error samples have been measured until now. The sum operators of ADEV (blue bracket pointing the samples no. 1, 4, and 7), TDEV (green bracket pointing the samples no. 1, 4, and 7 and light green bracket pointing the samples no. 1, 4, 7, and 10), and HDEV (red bracket pointing the samples no. 1, 4, 7, and 10) are not active now. The middle stage of the measurement and computation process is presented in Fig. 4. This time seven time error samples have been measured until now and the ADEV sum operator and TDEV internal sum operator are active from this instant. The operators of the overall sum of time deviation and the operator of the sum of Hadamard deviation are still not active. Figure 5 presents the stage of the process after the sample no. 10 has been measured. The ADEV operator is active and the sum of squares was updated using samples no. 10, 7, and 4. The TDEV internal operator is not active now – the internal sum is computed now and from this instant the overall sum operator (pointing four samples) is active. The HDEV sum operator (identical with TDEV overall sum operator) is also active from this sampling instant. The computation process for single observation interval only is presented in this example. The operations for successive greater observation intervals (greater n) have to be executed sequentially within one sampling interval [6, 7].

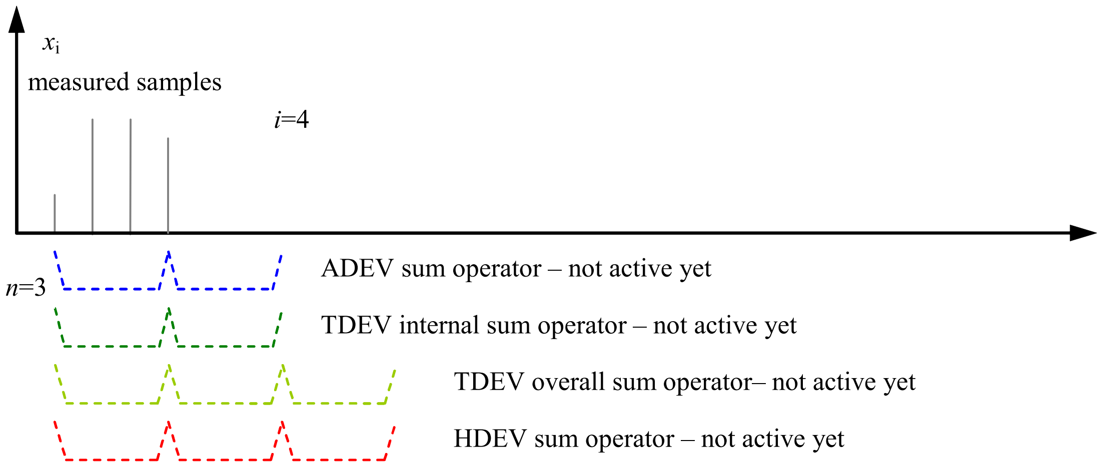

Fig. 3. Real-time ADEV, TDEV, and HDEV computation for observation interval $3\tau_0$, sample number 4 has been measured, computations are not active yet.

The brackets pointing the samples and distinguishing the operators for each parameter have symbolic meaning only. In the case of joint computation the samples needed for calculation of the parameters for the current instant are read out from the equipment memory at once, using one procedure involving three samples (indexed by i, $i-n$, and $i-2n$) at the early stages of the measurement process and four samples (additionally the sample indexed by $i-3n$) at the late stages. Therefore, the influence of the most critical issue – access to the measured data – on the calculation time within one sampling interval is reduced [6, 7]. In such situation we can expect, that the additional operations (next to HDEV computation) performed during joint computation (e.g. ADEV and TDEV) using the same set of samples are not critical burden for the calculation process.

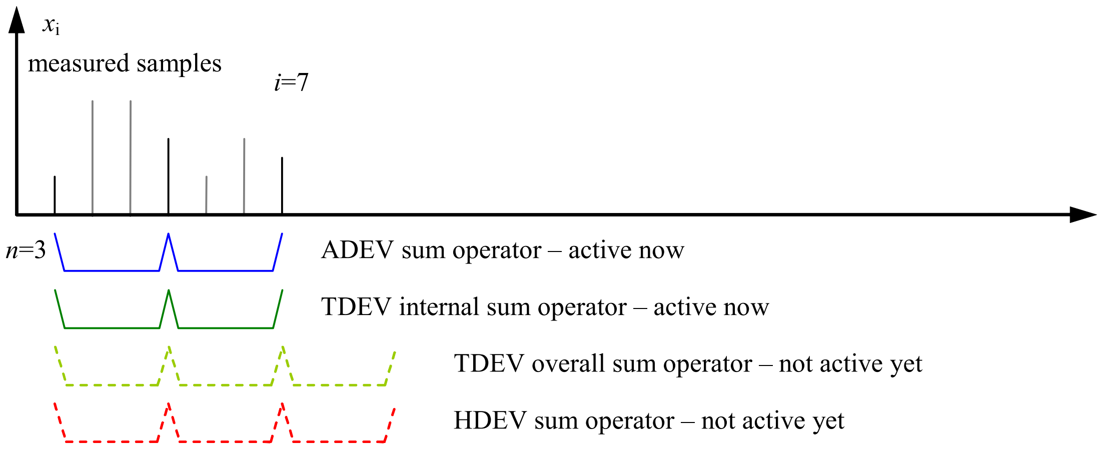

Fig. 4. Real-time ADEV, TDEV, and HDEV computation for observation interval $3\tau_0$, sample number 7 is measured, ADEV sum operator and TDEV internal sum operator are active now, TDEV overall sum operator and HDEV operator are not active yet.

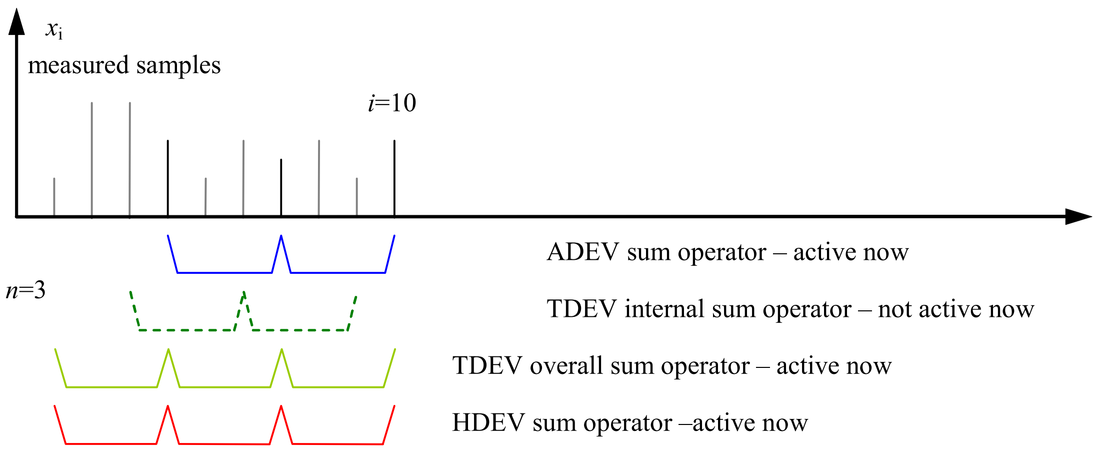

Fig. 5. Real-time ADEV, TDEV, and HDEV computation for observation interval $3\tau_0$, sample number 10 is measured, ADEV sum operator is active, TDEV internal sum operator is not active now, TDEV overall sum operator and HDEV operator are active now

COMPUTATION EXPERIMENT

The method of real-time computation of Hadamard deviation performed separately or jointly with Allan deviation or/and time deviation was tested in the experiment. The experiment was realized similarly as the tests of real-time ADEV and TDEV computation methods presented in [6, 7]. The calculations were performed off-line with the imitation of on-line work. The data sequence contains time error samples taken with the sampling interval τ_0=1/30 s during the time of 4000 s, representing white phase noise combining with frequency drift.

The calculations were performed for variable numbers of observation intervals, arranged in the logarithmic scale in a range between 0.1 s and 1000 s. The starting (smallest) observation interval was τ_{min}=0.1 s (n=3) and the longest observation interval was τ_{max}=1000 s (n=30000). The calculations were performed for 5, 10, and 20 observation intervals per decade.

Two personal computers with Intel Pentium IV 3.0 GHz and Intel Core 2 Quad 3.0 GHz microprocessors were used in the experimental tests. The maximum time used for calculation within one sampling interval was the observed quantity. We have assumed that this time cannot exceed the length of sampling interval τ_0=1/30 s = 0.0333… s = 33.3… ms.

Calculations were performed for HDEV only, HDEV jointly with TDEV or ADEV, and jointly for all three parameters. The time of computation for both computers is presented in Table 1 and Table 2. The values of TDEV and HDEV computed successively during the measurement are presented in the form of three-dimensional plots in Fig. 6 and 7.

Table 1. Time of computation using computer with Pentium IV 3.0 GHz

Parameters computed jointly	Number of intervals per decade		
	5	10	20
	t-max [ms]	t-max [ms]	t-max [ms]
HDEV	0.57	1.10	2.20
HDEV and ADEV	0.62	1.15	2.31
HDEV and TDEV	0.63	1.16	2.34
HDEV, TDEV, and ADEV	0.66	1.22	2.46

Table 2. Time of computation using computer with Intel Core 2 Quad 3.0 GHz

Parameters computed jointly	Number of intervals per decade		
	5	10	20
	t-max [ms]	t-max [ms]	t-max [ms]
HDEV	0.21	0.41	0.76
HDEV and ADEV	0.23	0.44	0.80
HDEV and TDEV	0.24	0.44	0.81
HDEV, TDEV, and ADEV	0.25	0.46	0.85

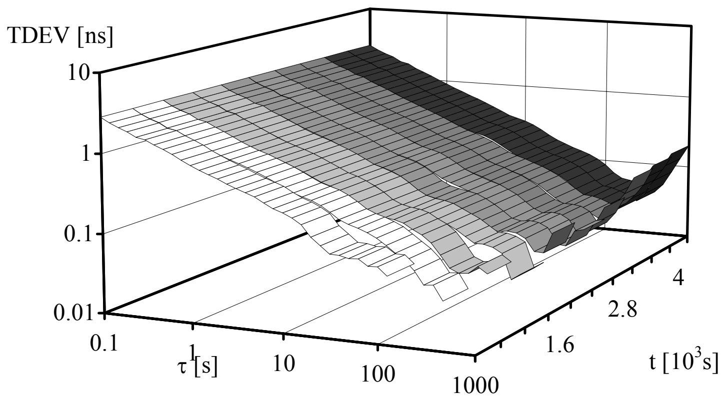

Fig. 6. Result of TDEV real-time computation successively obtained

The shape of the curves representing the results of TDEV computation (Fig. 6) shows the presence of the WPM noise and the influence of some additional process – frequency drift or phase noise of higher order. This influence can be recognized from the shape of the TDEV curve computed after 2000 s of the time error measurement. The shape of the HDEV curves (Fig. 7) shows the presence of the WPM noise only, so the additional process present in the time error sequence is the frequency drift. First segments of the TDEV and HDEV plots are shorter, because the parameters' values for longer observation intervals were not computed for these instants of the time error measurement process.

The time results obtained for both computers (Table 1 and 2) were satisfactory for all cases considered. Even the most time consuming case – computation of three parameters jointly for 20 observation intervals for decade – brought good result. The maximum time of operations performed for one sampling interval does not exceed the length of considered sampling interval 1/30 s. The application of computer with more modern microprocessor makes the computation time to be shorter. The results presented in the Table 1 and 2 confirm our expectation, that additional operations included to the real-time HDEV computation process does not burden the whole process of real-time computation.

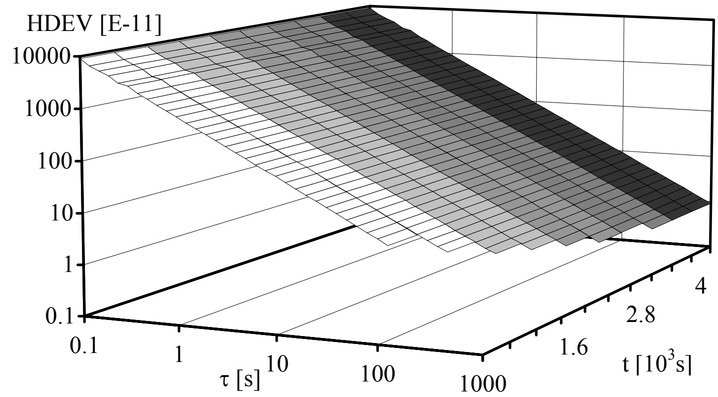

Fig. 7. Result of HDEV real-time computation successively obtained

The computation complexity of the parameters considered does not depend on the length of observation interval; the number of observation intervals analyzed simultaneously is the only limiting factor. Therefore, having limited computational capacities, we can choose wider range of observation intervals or greater number of observation intervals for one decade (resolution of the computation results on the scale of observation intervals). The results presented allow us to expect good results (maximum time not exceeding the sampling interval) for wider range and greater number of observation intervals than parameters considered in the experiment. We can also make the real-time computation of HDEV, TDEV, and ADEV for smaller value of sampling interval than the value of 1/30 s considered in the experiment.

CONCLUSIONS

The results of the experimental tests have proved the ability of the real-time computation of Hadamard deviation, Allan deviation, and time deviation performed jointly. The computation of the three parameters can be performed jointly simultaneously for numerous series and wide range of observation intervals as well as rather short sampling interval. Performing jointly the computation of HDEV, ADEV, and TDEV give us additional useful possibilities of timing signal analysis.

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